

## Is the Use of Analogy in Law Logically Correct?

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*Summary:* 1. The concept of analogy. — 2. Analogical judgement. — 3. Analogical arguments. — 4. Induction and analogy. — 5. Logic and analogy. — 6. The structural contradiction. — 7. Analogy in law. — 8. The moral of the story. — 9. Conclusion.

A concept of analogy does not exist — if by concept one means, as one usually does, a genus inclusive of all of its species. Indeed, there is no genus ‘analogy’ whose species allow themselves to be defined *per differentium*. Rather than a genus, we are dealing with a ‘family of concepts’, in the mathematical sense of the expression: from the point of view of ordinary logic, this is equivalent to saying that, with respect to their genus, these different concepts (or species) are only extrinsically alike. To set the question straight, we should above all distinguish analogical concepts, judgements, and arguments. They are not the same thing.

An ‘analogical concept’ (not to be confused with the aforementioned ‘concept of analogy’) is so named when its meaning cannot be defined in a univocal manner, but nor can it be considered an equivocal notion; in this respect, modern epistemology prefers to talk about ‘open concepts’ (for example, ‘mass’,

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\*This text is intended for the *Studi in memoria di Angelo Gualandi*.

Note — In itself, the line of reasoning of what follows is very simple. Since, however, it must be corroborated by many specifying or collateral remarks, it may not be entirely clear. Therefore, it seemed to me opportune to delineate the scheme in advance.

The use of analogy cannot in general be justified logically. That which holds in general must also hold for law. (Proposition A) — On the other hand, the use of analogy in law is indispensable. (Proposition B) — In the conjunction of A and B there is hidden a contradiction. (Proposition C) — A and B serve as premises, C as the conclusion.

The conclusion lends itself to two different evaluations, one negative and one positive. We choose the second. It is known that from a contradiction one may logically deduce anything. We therefore deduce, in a perfectly Kelsenian manner, the logical possibility of an anti-formalistic conception of law.

For the record, this is a thesis that the writer has derived from discussions with Angiolino Gualandi.

[*Translator’s Note:* The present text is an English translation of an article written by Enzo Melandri and published under the title ‘È logicamente corretto l’uso dell’analogia nel diritto?’ in the *Rivista Trimestrale di Diritto e Procedura Civile*, XXII, 1968, pp. 1–23.

A copy of the essay appeared with the same title in *Studi in memoria di Angelo Gualandi*, 2 vols., II, Urbino, Argalia, 1969, pp. 109–32.

Sections 7–9 of the essay were then reproduced with modifications and additions in §§ 103–104, ch. XIV, Part II of Melandri’s *La linea e il circolo. Studi logico-filosofico sull’analogia*, Bologna, Il Mulino (Collezione di testi e di studi. Filosofia), 1968.]

‘simultaneity’, ‘distance’, etc.): namely notions that are only definable in an implicit way, with respect to a particular gnoseological system, and that therefore may be perfected and redefined as knowledge progresses. But there is a much simpler example that is just as significant: all analogical concepts are metaphors, even if not all metaphors are analogical concepts.

Aristotle defines metaphor in terms of the improper use of a name, that is, a transfer of meaning. The transfer can be from genus to species, from species to genus, from species to species, or — by analogy (κατ’ἀναλογίαν).<sup>1</sup> We are interested in the final case. The example that Aristotle gives of the translation *per analogiam* is that one may call a cup a ‘shield of Dionysus’ and reciprocally a shield a ‘cup of Ares’.<sup>2</sup> This inevitably prompts a recourse to the proportional analogy, ‘the cup is to Dionysus as the shield is to Ares’ and to the consequences that derive from it. And this, in turn, is equivalent to saying that analogical metaphors — taken as such: that is, not those that exchange a part for a whole or vice versa (synecdoche), nor those that exchange a cause for an effect or vice versa (metonymy), but rather those that exchange a part for a part (catachresis), on the basis of a certain analogy — depend on the plausibility of a certain proportional relation: ‘*a* is to *b* as *c* is to *d*’. The question, it must be admitted, no longer relates to a ‘concept’ (a definition in terms of *genus et differentium specificum*), but it relates to a ‘judgement’. It is a matter of establishing what reasons we have for believing in the truth or validity of a certain proportional relation.

2. To make an analogical metaphor explicit, it is necessary to resort to similarity. Homer says that Agamemnon is a bull. This should be understood as a comparison: the excellence of Agamemnon among the Achaeans is similar to that which a bull exhibits, under certain conditions, among oxen.<sup>3</sup> But every similarity is founded on a proportional relation: ‘*a/b = c/d*’. In Aristotle’s example, ‘the cup is to Dionysus as the shield is to Ares’. Although it is expressed in comparative and not quantitative terms, the relation may be reduced equally well to a mathematical formula: ‘*c/D = s/A*’ (where *c* = cup, *D* = Dionysus, *s* = shield, and *A* = Ares). From the formula, one obtains ‘*c/s = D/A*’ (‘the cup is to the shield as Dionysus is to Ares’), as well as ‘*c·A = s·D*’ (‘the cup of Ares is like the shield of Dionysus’). This is as much as one can obtain from what Black calls, recalling Aristotle, the comparison view of metaphor.<sup>4</sup> That is not to say that all metaphors must be resolved into proportions, whether exact or only comparative. One need only observe that this rationalising exercise, which comprises ‘putting poetry into prose’, is a manifestation of the didactic ideology of our primary schools. Thus, we cannot renounce the comparative conception of metaphor and, at the same time, consider ourselves good educators or fit fathers. The rationalisation that we require is

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<sup>1</sup> *Poetica*, xxi 1457b<sup>6-9</sup>.

<sup>2</sup> *Poetica*, xxi, 1457b<sup>20-23</sup>; *Rhet.*, III, xi, 1412b<sup>35</sup>.

<sup>3</sup> *Iliad*, II, 480-84.

<sup>4</sup> Cf. M. Black, *Models and Metaphors: Studies in Language and Philosophy*, Ithaca, NY, 1962, ch. iii, § 4.

tantamount to recognising that similarity and proportion are not only concepts, but also and above all judgements, and that an analogical concept may only be obtained by abstraction from an analogical judgement.

Analogical judgements are distinguished from analogical concepts (or metaphors) due to the simple fact that, unlike the latter, they can be said to be true or false. Indeed, according to Aristotle, the virtual nature of the true-or-false is what distinguishes apophantic discourse (judgement) from discourse that has no such claim, since it is instead a direct or indirect expression of a subjective (emotional, evaluative, discretionary or perceptual) state.<sup>5</sup> For example, the proportion ‘ $8/4 = 6/3$ ’ represents a true judgement (a true proposition); while ‘ $8/4 = 6/2$ ’ is a false judgement (a false proposition). But why has this very simple fact, namely that analogy, similarity, and proportion have the logical status of judgements (or proportions), been repudiated for so long? To understand the reasons for this disavowal — they are not banal, and are yet to be fully overcome — it is above all necessary to say what distinguishes analogical, comparative or proportional judgement from conventional judgement, which is normally categorical. There are two very important differences:

(i) The first is that categorical judgements are always qualitative, while analogical judgements are quantitative, or at least regard proportion in a comparative sense. This means that the former is always univocally true or false, *tertium non datur*. According to the principle of the excluded middle, a thing either has or does not have a certain property or quality. Conversely, for the other type of judgement, it is essential that the qualities are ‘intensive’, that is, susceptible to gradation according to a criterion of more-or-less. Examples of its use are the comparative judgements of the form: ‘the more..., the greater...’ (direct proportion,  $y = x$ ); ‘the more..., the less...’ (inverse proportion,  $y = 1/x$ ); ‘much more... than...’ (geometric proportion,  $y = x^n$ ); ‘much less... than...’ (logarithmic proportion,  $y = \log_n x$ ), etc. At most, the principle of the excluded middle only holds true for the two extreme cases of proportion: ‘exact’ or mathematical analogy and ‘qualitative’ or absolute (non-comparative) analogy. For all the intermediate cases not only does the *tertium non datur* not hold, but it becomes much more coherent to make use of a ‘principle of included middle’. This means, in other words, to substitute a criterion of more-or-less-true — or — more-or-less-false for the criterion of true-or-false. We are familiar with this criterion. We use it constantly in our daily practices, and it is what Aristotle alludes to when, with regard to ‘distributive’ justice, he introduces ἀναλογία.<sup>6</sup>

Now, as the saying goes, if the practice of judging, ‘with all due proportion’ is the least that one may expect from any moderately wise and educated person, to this day the theory of this species of judgement, that is, the analysis of its formal structure, represents one of our most pressing *desiderata*. In fact, it leads directly

<sup>5</sup> *De Interpretatione*, iv, 17a<sup>1-5</sup>.

<sup>6</sup> *Ethica Nicomachea*, V, vi, 1131a<sup>29</sup>-b<sup>24</sup>.

to the problem of ‘plurivalent logics’: while on the one hand, it has been possible to establish their logical syntax, on the other, they still lack a congruent semantic interpretation, and it is not clear how there can ever be one.<sup>7</sup> Here we are faced with a veritable abyss.

(ii) The second difference is that categorical judgement always has two terms, ‘subject’ and ‘predicate’; while analogical judgement has four:  $a/b = c/d$ . Strictly speaking, one should say that it has five terms, that is four subjects ( $a$ ,  $b$ ,  $c$ ,  $d$ ) and a predicate (the ‘proportional function’) according to which ‘ $.../... = .../...$ ’. For more than two thousand years, taking Aristotle as the *terminus a quo*, the linguistic conditioning of logic induced the belief that predicates taking only a single subject (such as ‘ $x$  is  $P$ ’) were more primitive, underivable, and fundamental than predicates taking two or more subjects; and it is certainly not among the least of Russell’s merits to have introduced the notion of a ‘relational predicate’ (dyadic, triadic, ...,  $n$ -adic) into logic, in contrast with the ‘attributive predicate’ (monadic). This prejudice has ensured that proportion, which is a particular case of a double dyadic predicate (the equivalence of two relations) has been misunderstood time and again as an extrinsic union of two proportions or as a single, but imprecise relation of similarity.

This clarification serves better to distinguish analogical judgement and analogical inference. As we shall see, many of the confusions that persist in this regard are owed to the disregard of relational predicates and of the fact that proportionality is a particular relational predicate. It must be said at once, however, that this discovery does not take us very far either. We cannot generalise the notion of relational predicate and consider the normal attributive predicate a particular or even ‘degenerate’ case, in the mathematical sense,<sup>8</sup> without making the completion of every proposition and therefore judgement itself impossible. If in fact a monadic predicate is a degenerative case of a dyadic one, then by the same argument, one may say that the dyadic is such relative to a triadic, and so too the triadic..., *in infinitum*, until every predicate encompasses the entire universe of discourse. In this way, all propositions would inevitably degenerate into ellipses, allusion and anacoluthon.

3. From what we have said so far, it seems that the only guiding thread is offered by the notion of *proportion*, whether quantifiable or not. In the case that it is not quantifiable, we must content ourselves with a comparative judgement, more

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<sup>7</sup> The logical syntaxes of plurivalent logics — but from the point of view of a *bivalent* metalogic— were established for the first time by J. Łukasiewicz, *Elementy logiki matematycznej*, Warszawa, 1929: cf., the relevant bibliography in J.B. Rosser & A.R. Turquette, *Many-valued Logics*, Amsterdam, 1958. Regarding interpretation, we are equipped with two courageous, but perhaps premature pioneering works: A. Korzybski, *Science and Sanity: An Introduction to Non-Aristotelian Systems and General Semantics*, Lakeville, 1958 (1<sup>st</sup> ed. 1933); and G. Günther, *Idee und Grundriss einer nicht-Aristotelischen Logik*, I (‘*Die Idee und ihre philosophischen Voraussetzungen*’), Hamburg, 1959.

<sup>8</sup> Cf. e.g. S. Langer, *An Introduction to Symbolic Logic*, New York, 1953 (1<sup>st</sup> ed. 1937).

or less true depending on the case and according to the tolerance interval of the included middle. On this basis, analogical inference is nothing special: in fact, it is nothing but a particular case of predicate calculus, one that can be mediated by proportion, whether mathematically or only comparatively. However, tradition presents analogy as a separate scheme of reasoning or form of argument, distinct from both induction and deduction. Since this does not accord with the results of our analysis, we must reach some agreement over it before proceeding. There are at least two gross misunderstandings that require clarification.

(i) In general, Aristotle distinguishes between three species of inference: *deductive*, which goes from the general to the particular; *inductive*, which goes from the particular to the general; and *analogy*, which goes from the particular to the particular.<sup>9</sup> It is difficult to find expressed in so few lines, under the guise of apparent clarity, an accumulation of so many incongruities. And what about John Stuart Mill, who, after more than two thousand years, repeats the same definition? Analogy is the primary form of inference, since ultimately ‘all inference is from particulars to particulars’.<sup>10</sup> Aristotle’s example, then, very much merited the title of *paradeigma*.<sup>11</sup> It is concerned with dissuading the Athenians from going to war with the Thebans. We begin by observing that the Theban’s war against the Phoenicians was bad for both sides. Therefore, we compare the two wars: the Thebans’ war against the Phoenicians and the Athenians’ war against the Thebans. They have a common feature: they are wars between neighbours. Given that, by induction, all wars between neighbours are probably bad, one concludes that war between the Athenians and the Thebans will probably be similarly bad.<sup>12</sup>

It is not clear where the peculiarity of the argument lies. If the major premise is true (‘all wars with neighbours are bad’), the conclusion is given by deduction with a *modus darii*. If instead it is only probable, then the analogy becomes an induction reached by a single previously observed case. The difference between having observed a single case rather than ten thousand is doubtless notable, but, from a logical point of view, it is entirely irrelevant. Logic is concerned not with the truth of the premises, but only with the validity of the argument. Aristotle himself understood this perfectly well, when he observes that every proof (or demonstration) is also a syllogism (a deductively valid argument), but not every syllogism is a proof.<sup>13</sup> One can make the same kind of criticism when analogy is defined as an *argumentum a simili*.<sup>14</sup> one cannot infer anything from one particular to another, save by means of a general premise. If the latter is true, the conclusion

<sup>9</sup> *Analytica Priora*, II, xxiv, 69a<sup>13–16</sup>; *Rhetorica*, I, ii, 1357b<sup>25–30</sup>. It must be said, however, that the definition refers to the *paradeigma* (or *exemplum*), not *analogy*; however, we still consider the Aristotelian ‘paradigm’ as an exemplary case of analogical inference.

<sup>10</sup> *A System of Logic*, II, iii, § 4.

<sup>11</sup> See above, n. 9.

<sup>12</sup> *Analytica Priora*, II, xxiv, 68b<sup>41</sup>–69a<sup>11</sup>.

<sup>13</sup> *Analytica Priora*, I, iv, 25b<sup>30–31</sup>.

<sup>14</sup> Therefore, the Scholastics more opportunely called it *locus a simili*.

is given deductively and constitutes a ‘proof’, that is, an outright truth. If on the other hand the premise is not true, but only probable (by induction), then the conclusion is also ‘probable’ (in the ancient and mediaeval sense), namely, relative to the truth of the premises. The question of the probability of the premises represents an inductive problem, but not such that it serves to discriminate between induction and analogy. In such a case, analogy is an induction founded on the bare minimum, not another species of inference.

What sets Aristotle on the wrong path is the desire to try to distinguish between logic and rhetoric as forms of argument. According to this conception, rhetoric would be defined by the *enthymeme* and the *paradeigma*: the first being rhetorical syllogism and the second rhetorical induction.<sup>15</sup> With all due respect to Perelman,<sup>16</sup> this explication is absolutely unintelligible. Arguments cannot be divided into logical and rhetorical ones. Arguments are only logical. Whether their conclusions are cogent or not — once it is ascertained that they really are logical, that is, valid — depends on the truth of the major or minor premises and, therefore, also on the fact of having included all of the relevant information in them. It is clear that one can never be entirely sure of this, but the greater or lesser margin of uncertainty is not a sufficient reason to distinguish a new species of argument.

(ii) Aristotle founds induction on a special syllogism, the ‘syllogism by induction’: ὁ ἐξ ἐπαγωγῆς συλλογισμός.<sup>17</sup> If the two general premises are true, and not only probable, we have a perfectly valid deductive syllogism of the third figure in *modus darapti*. The condition of the truth of the premises, in an inductive argument, is satisfied by the ‘complete enumeration’ of all the particular cases.<sup>18</sup> And this requirement is justified in Aristotle by the fact that he identifies particulars with the ‘species’ and not, as we do, with the individual. An induction founded on the individual case can never be complete, since the enumeration always remains open (‘simple enumeration’). Instead the number of species contained in the genus may well be — and usually is — finite. Therefore, we do not have the same reason that Aristotle had to speak of a ‘logic of induction’. The expression is still in use and we do not intend to ban it; here we need only expose its fundamental ambiguity. According to the two senses of the genitive, the expression may mean two things: (a) that it is necessary to use logical reasoning in inductive procedures — and so what it says is true, but banal (every activity that requires a minimum of reflection likewise has its own logic: for example, the game of chess); or (b) that induction is founded on a peculiar ‘inductive’ logic, distinct from deductive logic and endowed with its own laws — and then what it says is very interesting, but

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<sup>15</sup> *Rhetorica*, I, ii, 1356b<sup>2-5</sup>.

<sup>16</sup> Cf. Ch. Perelman & L. Olbrechts-Tyteca, *Rhétorique et philosophie: Pour une théorie de l’argumentation en philosophie*, Paris, 1952; *Traité de l’argumentation*, 2 vols., Paris, 1958. Make no mistake, Perelman’s studies are worthy of the utmost consideration; we limit ourselves to highlighting an underlying inconsistency: there cannot be a logic of bad logic.

<sup>17</sup> *Analytica Priora*, II, xxiii, 68b<sup>15</sup> ss.

<sup>18</sup> *Op. cit.*, 68b<sup>28-29</sup>.

unfortunately false. Now, what holds for induction also holds for analogy, which is often considered to be a special form of induction.

What creates the confusion is the fact that inductive procedures may be expressed in logical formulas. In this way, the important, in fact decisive difference that lies between ‘formulation’ and ‘foundation’ slips away. For example, the laws of physics are also given mathematical expression; but we would not for this reason say that they are founded on mathematics. Given this, we may now say without fear of confusion what the difference is between the ‘logic’ of inductive inference and that of analogical inference.

4. According to Kant, induction and analogy are inferential modes; specifically, they are the only two possible *Schlussarten* of the faculty of judgement.<sup>18</sup> Let us remember that the latter, *Urteilstkraft*, is itself in this case the faculty of ‘reflective judgement’,<sup>19</sup> which goes from the particular to the universal and, in doing so, cannot be ‘determinative’, because it only has ‘subjective’ validity.<sup>20</sup> Now, the passage from particular to universal, under the jurisdiction of *reflektierende Urteilstkraft*, can happen in two opposite ways. Either one concludes from many things to all of those remaining, and then we have ‘induction’ according to the principle of generalisation: *Was vielen Dingen einer Gattung zukommt, das kommt auch den übrigen zu*; or, one concludes from many properties in common to all of those remaining, and then we have ‘analogy’ according to the principle of specification: *Dinge von einer Gattung, von denen man vieles Übereinstimmende kennt, stimmen auch in dem übrigen überein*. To express the difference between determinate and reflective judgement in another way, we note that induction and analogy are not *Vernunftschlüsse*; rather, they are only *logische Präsumptionen* or *empirische Schlüsse*. Therefore, to repeat Kant’s conclusion: *Eines in vielen, also in allen: Induktion*; while *vieles in Einem (was auch in anderen ist), also auch das übrige in demselben: Analogie*.<sup>21</sup> Here too, the particular mechanics of inflected language make the characterisation of analogy less clear, even though it is nothing besides the exact reciprocal of induction. Let us try to put them into formulas.

(i) Induction — Given that cases  $a_1, a_2, \dots, a_n$ , which have the property P (are instances of the case P), also have the property Q, we suppose that the new case  $a_{n+1}$ , by definition a case of P, although we do not know if it also has the property Q, also has the property Q. For example: if all Swedes observed until now were blond, one should expect that the next Swede we meet would also be blond.

(ii) Analogy — Given that cases  $a_1, a_2, \dots, a_n$ , which have the property P (are instances of the case P), also have the properties  $Q_1, Q_2, \dots, Q_k$ , we suppose that the new case  $a_{n+1}$ , by definition a case of P, granted that it also has the properties,  $Q_1, Q_2, \dots, Q_k$ , also has in common with the preceding cases ( $a_1, a_2, \dots, a_n$ ) an

<sup>18</sup> *Logik*, ed. Jäsche (1800), § 81.

<sup>19</sup> *Kritik der Urteilstkraft*, Einl., iv; *Erste Fassung der Einl.*, v.

<sup>20</sup> *Logik*, § 81.

<sup>21</sup> *Op. cit.*, § 84.

ulterior property, let us say  $Q_{k+1}$ . For example: if all Swedes observed until now were blond and long-limbed, one should expect that the next Swede we meet, granted that they will be blond, and long-limbed, will allow us to say something additional about Swedish people (I don't know — that they are tall, thin, or lanky).

One immediately sees that, aside from its linguistic trappings, analogy (that is, let us not forget, analogical inference) demands a further specification with respect to induction: that the observed instances, beyond belonging to the same case, also confirm the preceding inductions. In fact, one may only inductively infer a new property on this condition. In the post- and neo-Kantian literature, this has been called *Merkmalsinduktion*.<sup>22</sup> The point may lead us to think that induction logically precedes analogy. In reality, as Keynes has demonstrated, the delineated formula only establishes the conditions for a definition of 'positive' analogy.<sup>23</sup> If one eliminates the formula's limiting conditions, then such a case can always be made to hold. If the next Swede that we meet is not blond, we can always say that he has in common with other Swedes the property of being-(blond or not blond) and therefore infer from this the ulterior property of being-(long-limbed or not long-limbed), etc. In fact, analogy may also be 'negative' in Keynes's sense. From a logical point of view '+' and '-' are complementary signs and, therefore, equivalent. There is no reason to prefer one or the other. It is only from a practical point of view that positive analogies become preferable.

As we have seen, in this context the difference between induction and analogy amounts to the difference between 'extensional' generalisation (if an individual has a certain property, another will too) and 'intensional' generalisation (if two individuals have a certain property in common, they will also have another). Reduced to a qualitative form, in terms of individuals and properties, or denotation and connotation, both forms of non-deductive inference in turn risk degenerating into tautologies. Since every individual can be defined by a set of properties through 'characterisation', and every property by a set of individuals through 'abstractive classification', the difference between induction and analogy points to what we intend to define, whether it be a property or an individual. Then 'new individual' would be the inductive *definiens* of 'same property', just as 'new property' would be the analogical *definiens* of 'same individual'; and neither would say anything, except to better inform us about our linguistic usage. (The difference corresponds to the gnoseological one between 'physicalism' and 'phenomenalism', and is similarly tautological). Nor is this the worst of it. If we shift the focus from syntax to semantics, and then redefine the meaning of the terms as a function of the presupposed syntax, it is inevitable that everything, in the end, will appear to confirm the postulates with which we started, no matter how erroneous they are. To discover their limits, it is necessary to disassociate once again what has been

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<sup>22</sup> Cf. Drobisch, Wundt, Erdmann (see H. Hoffding, *Begrebet Analogie*, Kobenhavn, 1923; German trans., Leipzig, 1924).

<sup>23</sup> J. M. Keynes, *Treatise on Probability*, Part III ('Induction and Analogy'), ch. xx, § 2.



mistakenly associated from the start: language and reasoning. Here one reopens an inexhaustible problem, but this time, it may take us somewhat further.

5. The concept of ‘logical syntax’ is metaphorical and ambiguous; it is taken from linguistics and may assume different meanings according to which term is emphasised. After all, even the concept of logic is anything but univocal. We have already mentioned the aporetic condition that the theory of plurivalent logic finds itself in; we must add that research in that area has been carried out in order to give a less dogmatic systematisation of the foundations of logic. Normally one says that *an argument is logical when it is valid by virtue of its form alone*, namely, when it is analytic. However, recent developments in mathematical logic have shed light precisely on the ambiguity hidden in this definition of ‘analytic’: it may mean either ‘true by construction’, or ‘demonstrable’. And as Gödel’s second theorem shows, these two meanings are in no way synonymous.<sup>24</sup>

Therefore, we cannot bring inferential procedures into focus using the criterion of analyticity. We can reach a similar result by another means, which consists in disassociating the mathematics *of* logic, or *of* any species of reasoning (i.e. ‘calculus’), *from* language, and therefore comparing different calculi directly in order to highlight affinities and divergences. A calculus in fact represents that which is mathematical in logic, and not linguistic – as in every other form of reasoning: the ‘mathematics of logic’ is entirely different from the ‘logic of mathematics’. Now, if we grant that the mathematics of generally accepted, valid inferences (not only deductive, but also inductive and analogical) reveals the presence of at least two calculi, which are not only distinct, but also alternatives, then on the basis of this microstructural contradiction, it will be easy to give an account of other contradictions, and in particular of those that, at a macrostructural level, lead us on occasion to speak of the ‘crisis of foundations’ of this or that science.

In fact, it is possible to demonstrate the coexistence of two such calculi in every reasoning activity, whether ordinary or scientific. Here we shall omit the analytical work in order immediately to present its results.<sup>25</sup> After all, we are dealing with a matter that is also intuitively accessible. Let us start with two examples: the first may be taken from calculating machines. According to their mode of operation, these may be divided into two great complementary classes: one is formed by those that function according to the ‘digital principle’, which are called *numerical machines* or *digital computers*. They are the ones that compute beginning from discrete units, each univocally orientated towards ‘yes’ (1) or ‘no’ (0), and whose result is nothing but the integral summation. (Office adders also belong to this class.) The other class is formed by those machines that function according to the ‘proportional principle’, and these are called *analogy machines* or *computers*. In this case, both the input data and the result are always more or less approximate, since, as a rule, proportion excludes exactitude. (The common slide

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<sup>24</sup> This is not the place to enter into the question: an excellent presentation with the minimum of technicalities is E. Nagel & J. R. Newman, *Gödel’s Proof*, New York, 1958.

<sup>25</sup> This will be given elsewhere.

rule also belongs to this class).<sup>26</sup> The central nervous system provides another example. The brain, it seems, also operates on the basis of two distinct principles, one digital and one analogical. The first is connected with the structure of the nerve cell, the neuron, which is in a state either of excitement or rest: *tertium non datur*. The second is rather connected with the nerve fibres, in which single pulses may be regulated according to a ‘frequency modulation’, which is proportional and not all-or-nothing. The utility of a system of two complementary principles is evident: one of them is precise and analytic, but slow; the other is rapid and synthetic, but approximate.<sup>27</sup> Generalising, one may say that the digital principle corresponds to the *logical* calculus (in the strict sense), while the analogical one corresponds to the *proportional* calculus. We shall give a synoptic table of the characteristics that differentiate the two calculi, both in themselves, and in their normal reasoning application, whether ordinary or scientific:

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<sup>26</sup> Cf. e.g. N. Wiener, *Cybernetics: Or Control and Communication in the Animal and the Machine*, New York-London, 1948, p. 117; *The Human Use of Human Beings: Cybernetics and Society*, 1954 (1<sup>st</sup> ed. 1950), pp. 64, 151.

<sup>27</sup> Cf. e.g. W. Wieser, *Organismen, Strukturen, Maschinen: Zu einer Lehre vom Organismus*, Frankfurt am Main, 1959, pp. 88-89, 103-106.

### Logical Calculus

a) The principle of *all-or-nothing*, of *bivalence* (true-or-false), of the *excluded middle*. Plurivalent logics are subsumed under a bivalent meta-logic; the same holds for probabilistic logic.

b) The principle of *elementary identity*: given two reciprocally contradictory predicates, *P* and not-*P*, any individual *x* necessarily has one or the other; that which does not satisfy this condition is not an *individual*.

c) The principle of *extensionality*: every function (class, relation di-, tri-, ..., *n*-adic) is defined by its elements or dyads, triads, ..., *n*-ads of elements (individuals).

d) The principle of *discreteness*: the elements (individuals) are not connected with one another except by *external* relations, such that their intrinsic nature is not modified.

e) The principle of finitude: the number of individuals is plural, but not infinite (except potentially); every set or collection has to be denumerable.

### Proportional Calculus

a') The principle of continuous *gradation*, of *dipolarity* (between true and false), of the *included middle*. Plurivalent logics do not have an overriding meta-logic; the same applies to probabilistic logic, which admits of a direct, objective interpretation.

b') The principle of *functional identity*: the individual in itself does not have identity, except as a value *x* that satisfies the conditions required by the function it depends on; elementary identity is nothing but a particular case of *function*.

c') The principle of *intensionality*: every element (individual) is defined by abstraction from a classificatory function or relation, as a value *x* that either does or does not satisfy its conditions.

d') The principle of *continuity*: all relations are *internal*; between every two elements, however proximate, there is always room for a third.

e') The principle of infinity: it is not necessary that the number of individuals be plural; if it is, then the concept of actual infinity holds, which is reflexive and non-denumerable.<sup>28</sup>

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<sup>28</sup> An exhaustive illustration of these theses would entail too long a digression. As has been said, this shall be given elsewhere. We trust, however, that it is perspicuous at least in spirit. Once the kernel of the question is grasped, it should be clear how the contraposition of 1-5 and 1'-5' (in substance, if not in form) repels the critiques of the most rigid logical neo-positivism.

6. The best demonstration of the existence of the two calculi and of their difficult cohabitation is provided by the history of mathematics. Let us explain: mathematics is the inventory and attempt to rationalise all calculi in use. Perhaps the plural collective *mathematica* (*mathematiques*, mathematics) best indicates the question that concerns us. In fact, the ‘theory of functions’ is more analogical than logical; meanwhile, exactly the opposite holds for the ‘theory of sets’.<sup>29</sup> The historical crises of the ‘foundations of mathematics’ (Zeno, the irrationals, the infinitesimals, Cantor, Russell) become easily comprehensible on the basis of the contrast between two alternative species of calculus. Logic and analogy are the two poles between which the drama (or dialectic) of the progress of rational thought unfolds.<sup>30</sup> The contrast between the two calculi is exemplified by the problem of irrational numbers. If one allows the theory of proportions to include numbers too, then the following antinomy is immediately revealed: a number is defined by the fact that it is a collection of units (in the ‘logical’ sense). Now a collection of units has to be, among other things, either even or odd (= not even): *tertium non datur*. But if one applies proportions to numbers, the proportion  $2/x = x/1$  also becomes legitimate, from which  $x = \sqrt{2}$ . And it can be demonstrated that the square root of 2, if it is a number, can be neither even nor not even; therefore, if it is a number, then it is not a number (it is not a collection of elements). This aporia was already known to Plato.<sup>31</sup>

The example of mathematics goes to demonstrate the presence of a latent contradiction in the microstructure that may or may not become patent at the molar level. The history of mathematics demonstrates that every so often latent contradictions appear in the macrostructure too, and furthermore that during the intervals between the crises they can be traced back more or less easily to a dialectic of complementarity. The example of mathematics seems relevant under two aspects: (a) according to the broad sense by which we have characterised its role, namely, as the science of calculi, it is mathematics, and not logic – which represents the interests of a special calculus – that can tell us something about our mode of reasoning; (b) the presence of a contradiction – even if it is motivated by endless reconsiderations – is not in itself a sufficient condition for declaring the failure of rational thought, as it is (and must be) in logic. In other words, mathematical rationality is much broader than logical rationality. And this is explained only if we admit a plurality of calculi into mathematics. Furthermore, it is necessary to underline that mathematics, logic and analogy (and therefore every form of rational thought that is not strictly logical) each have a different function and therefore a different relative weight. Logical concerns intervene in the ‘context of justification’,

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<sup>29</sup> Cf. e.g. O. Becker, *Grösse und Grenze der mathematischen Denkweise*, Freiburg u. München, 1959.

<sup>30</sup> Cf. e.g. G. Polya, *Induction and Analogy in Mathematics*, Princeton, NJ, 1954.

<sup>31</sup> *Leges*, VII, 819c<sup>7</sup> ss.

while the analogical calculus is better adapted to a ‘context of discovery’.<sup>32</sup> Given the strict correlation between induction and analogy, it is easy to understand – despite the best efforts of Frege, Peano, and Russell – why a complete logical justification of ‘mathematical induction’ is not possible; conversely, one may say that it is the same concern of providing logical justification that produces ‘crises of foundation’.<sup>33</sup>

Before moving on to analogy in law, two clarifications are needed: if for no other reason than to render the μετάβασις εἰς ἄλλο γένος less abrupt. Firstly, it is not the case that the relationship between the microstructural contradictions and the macrostructural contradictions that exist in mathematics need also hold in law. According to Kelsen’s conception, law is a formal science like logic and mathematics. But it is doubtful that its formalism can ever reach the level of precision that, in mathematics for example, produces time and again true and proper crises of foundation. It appears more reasonable to expect that, in law, logic always knows how to subordinate the contrasting tendencies of analogy to its own requirements.

Secondly, however, it is necessary to recognise that the reciprocal can also hold. It is not certain that what is the case for mathematics is also the case for law; but nor is it certain – for exactly the same reasons – that it is not the case. Naturally, if this is true – if, that is, analogy can offer a legitimate counter to logic – then this must be demonstrated by macrostructural arguments, namely, with arguments internal to law and to its history (admitting that its history is exegetically relevant). And then the reference to mathematics and to the relationship between microstructure and macrostructure should be intended, in turn, in an analogical sense: if it so happens that in juridical exegesis analogy enters into conflict with logic, one should not jump to conclusions about the degeneration of exegesis into irrationality; the example of mathematics teaches us that logic does not cover the entire field of rationality, and that therefore the task of a truthful rationalisation, one that wants to be more than a justification of what is already given, known, and judged, remains open.

7. The historical premises that have determined the contemporary development of analogy in law and the methodological difficulties that until now have invalidated its correct use form the theme of Bobbio’s well-known work.<sup>34</sup> On the face of it, the problem that analogy is called upon to solve is very simple. The question is connected with the equivocal interpretation that may be given to a law when the case is not univocally individuated by the law’s content. When the interpretation is univocal, there is no problem, and the task of the judge is limited

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<sup>32</sup> The distinction between context of justification and context of discovery is taken from H. Reichenbach, *Experience and Prediction: An Analysis of the Foundations and the Structure of Knowledge*, Chicago-London, 1961 (1<sup>st</sup> ed. 1938), pp. 6-7, 381-382.

<sup>33</sup> P. H. Nidditch, *The Development of Mathematical Logic*, London, 1962, esp. pp. 71-72.

<sup>34</sup> Bobbio, *L’analogia nella logica del diritto*, Torino, 1938; of which the first part is dedicated to history, while the second is dedicated to the theory of juridical analogy.

to verification. As we know, the judicial operation may be configured under the species of a *modus darii* syllogism (or of a ‘subsumption’). The law constitutes the (universal) major premise; the verification, the (particular) minor premise; and the sentence provides the conclusion (also particular). This, however, is a limit-case. In reality, laws cannot foresee all of the cases to which they will be applied, but only some of those that are recurrent and typical or, simply, those particular cases that the legislator has in mind when instituting the norm.<sup>35</sup>

Usually, therefore, the application of laws demands a preliminary work of interpretation, which is the responsibility of the judge. The effect of the interpretation on how the syllogism (that is, the sentence) is formulated may vary from a minimum to a maximum; generally, however, it never fails to make its mark. Consequently, one should not consider interpretation an extrinsic complement of the juridical order in act, but rather its essential, supplementary factor. Its contribution pervades the whole order, even if it is not distributed equally across all the laws in which it finds expression. It will be much more relevant, for example, in the case of laws that prove to be inadequately formulated. The terms in which these have been drawn up, we may suppose, express a certain defect of approximation: either they are too general with respect to their referent, or vice versa, they are too particular; thus, they are ambiguous one way or another. In these cases the judge’s task is that of interpolating gaps (or, as we say, inserting a stopgap), that is, to close the loopholes in the juridical order. To this end, he must use analogy, which thus becomes the methodologically indispensable, interpolative vehicle for obtaining the uniform, complete and fully regulative *extensio legis* of the legal order. This use of analogy, specified as *analogia legis* (and not *iuris*), is not only generally permitted without restriction, but is actually recognised as a universal, and not only a juridical, canon of interpretation.<sup>36</sup>

Up until now, the solution of the problems does not appear to present any particular difficulty. However, a different case arises when legal imperfections are not intrinsic, but rather due to changes in the historical context in which they are applied, that is, to put it mildly, due to a discrepancy that opposes a ‘legal’ state to a ‘real’ one. We do not immediately have to jump to the phenomenon of the obsolescence of the law. As we shall see, the inverse phenomenon, under the same name, may also exist: for example, that of constitutional rights that pre-date the law and confront it with a prior order. Whatever the case may be, it seems clear that the application of analogy must now resolve a much more complex problem, which, to use structuralist terms, is no longer just ‘synchronic’, but also and above

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<sup>35</sup> The thesis is well known, but it is worth seeing its exact formulation in the terms of modern symbolic logic: cf. U. Klug, *Juristische Logik*, II. Berlin, Göttingen u. Heidelberg, 1951, § 5, 9 (pp. 58–63).

<sup>36</sup> On analogy as the universal canon of every hermeneutics, cf. W. Dilthey, ‘Die Entstehung der Hermeneutik’. Ges. Werke, V, pp. 317–38.

all ‘diachronic’.<sup>37</sup> It is equally clear that the judge’s task grows in importance commensurately. He can no longer limit himself to interpolating the gaps *between* the laws, but must extrapolate something new *from* the laws: that is, he must take on the risk of prolonging, as though it were a tangent, what is metaphorically called the ‘will of the legislator’. This implies a much more complex judgement, which cannot be attributed to a formal syllogism. Nor is it a minor inconvenience. Even if the judge is unaware of it (or, better put, even if he is induced, for entirely comprehensible reasons, to ‘repress’ it), it is clear that such a judgement implies a historical evaluation of tendencies at work within history. Whatever the case, it is a fact that the extrapolation we are alluding to does not simply fill in gaps, but amounts to an effective creation of law. In this loaded, risky and problematic use, analogy is called on to give an ‘intensive’, and not only an ‘extensive’, interpretation of norms; therefore, to the extent that it assumes this function, it is specified as *analogia iuris*. The gap that must be filled, in other words, is ‘subjective’ and not ‘objective’: it is a gap of the law, and not in the laws.

Now, to borrow from Savigny, the juridical problem of analogy appears to concern neither more nor less than this: the legitimacy of its use in function of *Rechtsfindung*, namely, of the creation of law or of a criterion for its intensive interpretation; and this is in opposition to the already acquired function of *Gesetzesauslegung*, namely, of the interpolation of legal gaps or of the extensive criteria for the interpretation of laws.<sup>38</sup>

8. The difference between the two uses of analogy seems to be a radical one. Even the terminological distinction, if not exactly ancient, should nevertheless be deemed ‘classical’.<sup>39</sup> On the one hand, there is the *analogia legis*, which we imagine proceeds deductively, insofar as it follows the univocal principle of the *eadem ratio*; in any case, since it is an interpretation *ad supplendum*, the margin of arbitrariness is always greatly reduced. On the other hand, there is the *analogia iuris*, which is synthetic and not merely analytic. It institutes an interpretation *ad intelligendum*, which is enough to transgress the pure and simple objectivity of the law. The proportional average between the two species of analogy, ‘objective’ (legal, univocal, extensive, interpolative, and analytic) and ‘subjective’ (juridical, equivocal, intensive, extrapolative, and synthetic), is given by the following principle: *ubi eadem ratio, ibi eadem iuris dispositio*. Now this may have passed in the jurisdictional context of ‘Roman law’ or even of natural law ideology broadly construed. If one presupposes that the juridical order possesses an ideal, static

<sup>37</sup> On the pertinence of the questions regarding law raised by contemporary structuralism, both linguistic and anthropological, cf. Ghezzi’s essay, ‘A proposito di “strutturalismo” e metodo dell’indagine giuridica’ [Regarding ‘Structuralism’ and Method in Juridical Investigation], in this review: *Studi in memoria di Angelo Gualandi*, 2 vols., II, Urbino, Argalia, 1969, 1966, pp. 1411–27.

<sup>38</sup> Bobbio, op. cit., pp. 6ff., 66; for the critical history of this approximate attribution of paternity see *ivi*, p. 70.

<sup>39</sup> Op. cit., Part I (pp. 7–78).

nature, then the passage from the objective criterion of the *eadem ratio* to the logically weaker one of the *similis ratio* – which is at least in part subjective and therefore stands under the ‘induction’ rather than the deduction of law – does not present us with any prohibitive difficulties.<sup>40</sup>

But in Savigny, one notes a shift of emphasis that insists less on the logical moment of differentiation and more on its historical, psychological and broadly speaking concrete moment. The criterion of the *similis ratio*, insofar as it is a factor in a positive, historical, and creative *Rechtsfindung*, must necessarily distance itself from the criterion of the purely tautological factor of the *Gesetzesauslegung* founded on the *eadem ratio*.<sup>41</sup> In his ostensible conservatism, Bobbio’s intervention is decisive on this point. Having posed himself the ‘logical’ problem of analogy in law, he cannot but reach restrictive conclusions. Analogy is only justifiable insofar as it serves as an analytic interpretation and not the creation of a new law. Even if it does not limit itself to the interpolation of laws – as it also comprises a certain extrapolation of the law itself – analogy remains logically unjustifiable nonetheless, save the extent to which, proceeding analytically, it renders the implicitly presupposed rationality of the juridical system more explicit – and therefore, only if it functions as an instrument for legal objectification.

There is every reason in the world to think that endeavouring to read between the lines is not only *osée* [daring], but forever objectionable. At this point, however, I cannot help but do so *coûte que coûte* [no matter what the cost]. The distinction that Bobbio establishes between the two uses of analogy coincides with neither the classical distinction, nor with Savigny’s distinction. In fact, both of the latter presuppose that the juridical order already represents a coherent system, even if they go on to evaluate the ‘anomalous’ function of analogy differently: in one case, to reinstate the preceding legality; in the other, to produce a new legal system. For both positions, it is taken for granted that the coherence of the system is independent of the use, legitimate or otherwise, of analogy: except that in one case this coherence is located in the past, while in the other it may also be assigned to the future; analogy can restore the system’s coherence, or instigate it; however, it cannot produce it, because analogy is given separately from it. But, if I am not mistaken, for Bobbio the position is reversed. Analogy is neither a means for restoring the system, nor a means for transitioning from one system to another. Instead it is the distinction between the two species of analogy that, however understood, founds the *system* as a result of analytical restriction.<sup>42</sup> That is: for the preceding conceptions the coherence of the juridical system holds as a presupposition, and analogy is evaluated on this basis; while for Bobbio it is the different uses of the juridical analogy that guarantee – or fail to guarantee – the coherence of the system.

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<sup>40</sup> Op. cit., pp. 103–104.

<sup>41</sup> Op. cit., pp. 66–70.

<sup>42</sup> Op. cit., pp. 96–97; 103–104; 132–34.



We already know that, at the microstructural level, analogy and logic represent two incongruent modes of reasoning (§§ 5-6). It is not a given that this must also hold at the macrostructural level, but neither is it granted that it cannot hold. Bobbio's juridical philosophy, by making the legitimacy of analogy the principle of the legitimacy of the juridical system, establishes at one and the same time the tautology of the system and its limits. It is useless to insist on this point: it is in itself tautological and cannot take us any further. We know, however, that the criterion of the *eadem ratio*, applied to analogy, is not logically justifiable. In fact, it presupposes a proportional calculus, which in principle is in contrast with the strictly logical one, as demanded by the juridical definition of each case. This state of things is only contradictory in a virtual sense. In fact, as Bobbio demonstrates, one can always 'define' a type of analogy that satisfies the criterion of analyticity. But then, one is dealing with a criterion of analyticity which is, so to speak, 'postulatory', that is, relative not to that proportional calculus of which analogy is a direct expression, but to a special, restrictive interpretation of it, which is not justifiable on the basis of its intrinsic principles. Therefore, the appearance of a contradiction at the macrostructural level of the logic of law, even though it can be contained each time by resorting to special interpretative postulates, can never once and for all be severed at its root. The very *ad hoc* character of the procedure that rationalises analogy also serves to maintain indefinitely analogy's intrinsic, virtual capacity to contest any logic. What matters is that we recognise that analogy can contradict logic precisely 'by virtue of its form'.

9. This purely formal conclusion is also supported by extrinsic considerations. Examining the same problem from the point of view of modern logic, but keeping in mind the requirements of juridical procedure, Klug starkly concludes in favour of a criterion of *similis ratio*, in direct contrast with Bobbio's 'analytical' thesis.<sup>43</sup> Klug recognises that modern logic offers no opportunity for justifying analogy, whether subjective or objective. We have seen that the illogicality of analogy depends on two factors of a non-formal character: (a) the choice, predilection, or elevation to a paradigmatic status of a particular calculus, the strictly 'logical' one; (b) the exclusion, subsumption, or in any case the subordination to the λόγος κατ' ἐξοχήν of every other possible calculus or interpretation, with the aim of avoiding 'logical' contradictions. Therefore, Klug, who confuses illogicality with irrationality, is led to erroneous conclusions. In any case, here we underline the consequences that bind those who do not accept our way of posing the problem. For Klug, by being simultaneously a *necessarium* and an *irrationale*, analogy comes to be a part of praxis and not theory; or, better said, it falls within the sphere of synthesis and induction.<sup>44</sup>

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<sup>43</sup> U. Klug, *Juristische Logik*, cit., § 9 ('Der Analogieschluss'), pp. 98-126.

<sup>44</sup> Op. cit., p. 126.

A more concrete validation is provided by Tarello's recent work, which is dedicated to trade union law.<sup>45</sup> The Italian constitution of 1948 grants to every citizen the right to work (art. 4), whether in anticipation of or in contrast to existing legislation. Modern labour law develops on the basis of this lacuna or contradiction. As Tarello says, the right to work develops beginning from a 'legislative deficiency': and everyone sees how far this notion corresponds to the classical idea of a 'legislative gap'.<sup>46</sup> In this case *analogia iuris* is founded on the constitution itself, and the labour lawyers are left with the difficult task of legitimising it as an institutional *analogia legis*, even if this may provoke conflicts, tensions and contradictions with the juridical order already in force and its by now canonical interpolations. As Tarello observes, one can glimpse a macroscopic sign of such a difficult coexistence in the opposition, primarily methodological, and therefore on proper reflection also ideological, that is emerging between civil law doctrine and labour law doctrine: the first has now been caught up in a problem of exasperating interpretative formalism; while the second is not only open to the most pressing psychological, sociological and political questions of the present moment, but also (I would add) open to the historical and existential adventure of our times.<sup>47</sup>

We have attempted to demonstrate by formal considerations that logic, (a) if intended in a strict sense, has no right to hegemony over rational thought; and (b), if intended in a broad sense, it must also justify analogy, and, in the extreme case, even contradiction. For those who from the beginning have embraced the theses of Hegelianism, or to be a little more up to date, of Hegelian-Marxism (in the manner of Horkheimer and Adorno, but also in part Marcuse) this conclusion will undoubtedly be thought a foregone conclusion, if not outright banal. But, for those who approach the comprehension of Hegelian-Marxist dialectics by way of a so-called centripetal movement — that is, from the outside, arduously, and by overcoming many scruples, of which perhaps not all are misplaced — it is a conclusion that may be of some interest. The moral of the story, about which there should be no doubt, concerns everyone equally. Analogy in law does not only admit a conservative interpretation, but also a progressive one. It is precisely the progressive jurist that must make use of it, and consciously so: with the aim of inserting contradictions into a juridical order that rationalises existing power and serves as its ideological foil, by corroding it from the inside.<sup>48</sup> Analogy can also be subversive.

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<sup>45</sup> Tarello, *Teorie e ideologie nel diritto sindacale*, Milano, 1967.

<sup>46</sup> Op. cit., p. 25.

<sup>47</sup> Op. cit., p. 25.

<sup>48</sup> I cannot say whether Angiolino Gualandi would have accepted this conclusion. In his time (before 1965), I would rather say that he would not have; but, after all, at that time I would not have known how to put it to him with the same conviction as now. Making use of the counterfactual, I would venture to say that today he would have at least accepted it, if not exactly shared it. *Edel genug war er zum Widerruf!* — as Nietzsche says, referring to another who died equally young (*Also sprach Zarathustra*, I, 'Vom freien Tode').